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Even vertex odd mean labeling of graphs obtained from some graph operations

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ABSTRACT

A graph with p vertices and q edges is said to have an even vertex odd mean labeling if there exists an injective function $f : V(G) \to \{0, 2, 4, \ldots, 2q-2, 2q\}$ such that the induced map $f^* : E(G) \to \{1, 3, 5, \ldots, 2q-1\}$ defined by $f^*(uv) = \frac{f(u)+f(v)}{2}$ is a bijection. A graph that admits an even vertex odd mean labeling is called an even vertex odd mean graph. Here we investigate the even vertex odd mean behaviour of some standard graphs.

Keywords

even vertex odd mean labeling, even vertex odd mean graph.

2010 Mathematics Subject Classification Number. 05C78.

1. INTRODUCTION

Through out this paper, by a graph we mean a finite undirected simple graph. Let G(V,E) be a graph with p vertices and q edges. For notation and terminology, we follow [4].

The graceful labeling of graphs was first introduced by Rosa in 1967 [1] and Gnanajothi introduced odd graceful graphs [3]. The concept of mean labeling was introduced and meanness of some standard graphs was studied by Somasundaram and Ponraj [7]. Further, some more results on mean graphs are discussed in [6, 8]. A graph G is said to be a mean graph if there exists an injective function $f : V(G) \to \{0, 1, 2, \ldots, q\}$ such that the induced map $f^* : E(G) \to \{1, 2, 3, \ldots, q\}$ defined by $f^*(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ is a bijection.

In [5], Manickam and Marudai introduced odd mean labeling of a graph. A graph G is said to be odd mean if there exists an injective function $f : V(G) \to \{0, 1, 2, 3, \ldots, 2q-1\}$ such that the induced map $f^* : E(G) \to \{1, 3, 5, \ldots, 2q-1\}$ defined by $f^*(uv) = \left\lceil \frac{f(u)+f(v)}{2} \right\rceil$ is a bijection. The concept of even mean labeling was introduced and studied by Gayathri and Gobi [2]. A function f is called an even mean labeling of a graph G if f is an injection from V(G) to the set $\{2, 4, 6, \ldots, 2q\}$ such that when each edge uv is assigned the label $\frac{f(u)+f(v)}{2}$, then the resulting edge labels are distinct. A graph which admits an even mean labeling is said to be even mean graph.

A graph G is said to have an even vertex odd mean labeling if there exists an injective function $f : V(G) \to \{0, 2, 4, \ldots, 2q-2, 2q\}$ such that the induced map $f^* : E(G) \to \{1, 3, 5, \ldots, 2q-1\}$ defined by $f^*(uv) = \frac{f(u)+f(v)}{2}$ is a bijection. A graph that admits an even vertex and odd mean labeling is called an even vertex odd mean graph [9].

In this paper, we study the even vertex odd meanness of some graphs obtained from some graph operations such as vertex identification, edge identification and splitting.

2. MAIN RESULTS

The dump bell graph $C_m @ C_n$ is a graph obtained by joining two cycles of length m and n respectively by an edge.

Theorem 2.1. The dump bell graph $C_m @ C_n$ is an even vertex odd mean graph for positive integers m, $n \equiv 0 \pmod 4$.

Proof. Let $u_1, u_2, \ldots, u_m$ and $v_1, v_2, \ldots, v_n$ be the vertices of the cycle C_m and C_n respectively so that u_{m-1} is adjacent to v_1. Define $f : V(C_m @ C_n) \to \{0, 2, \ldots, 2(m+n)+2\}$ as follows:

$$f(u_i) = \begin{cases} 2i-2, & 1 \le i \le \frac{m}{2} \text{ and i is odd} \\ 2i+2, & \frac{m}{2}+1 \le i \le m \text{ and i is odd} \\ 2i-2, & 1 \le i \le m \text{ and i is even} \end{cases}$$

and $$f(v_i) = \begin{cases} 2m+2i, & 1 \le i \le \frac{n}{2} \text{ and i is odd} \\ 2m+2i+4, & \frac{n}{2}+1 \le i \le n \text{ and i is odd} \\ 2m+2i, & 1 \le i \le n \text{ and i is even.} \end{cases}$$

The induced edge labeling f^* is obtained as follows:

$$f^*(u_iu_{i+1}) = \begin{cases} 2i-1, & 1 \le i \le \frac{m}{2}-1 \\ 2i+1, & \frac{m}{2} \le i \le m-1, \end{cases}$$

$f^*(u_mu_1) = m-1$,

$$f^*(v_iv_{i+1}) = \begin{cases} 2m+2i+1, & 1 \le i \le \frac{n}{2}-1 \\ 2m+2i+3, & \frac{n}{2} \le i \le n-1, \end{cases}$$

$f^*(v_nv_1) = 2m+n+1$

and $f^*(u_{m-1}v_1) = 2m+1$.

Thus f is an even vertex odd mean labeling of $C_m @ C_n$. □

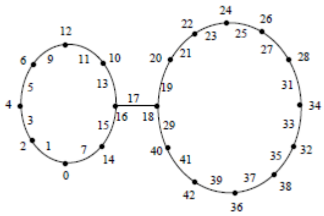

Figure 1. An even vertex odd mean labeling of $C_8@C_{12}$.

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The graph $C_m \ominus C_n$ is obtained by identifying any one of the edge of cycle C_m with an edge of cycle C_n.

Theorem 2.2. $C_m \ominus C_n$ is an even vertex odd mean graph for all even integers m, $n \geq 4$.

Proof. Let $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$ be the vertices of the cycles C_m and C_n respectively.

Case (i). m, $n \equiv 0 \pmod 4$ with $m \leq n$.

Let the identifying edge be $u_{m-1}u_m$ and v_1v_2.

Define $f : V(C_m \ominus C_n) \rightarrow \{0, 2, 4, \ldots, 2(m + n - 1)\}$ as follows:

$$f(u_i) = \begin{cases} 2i-2, & 1 \leq i \leq \frac{m}{2} \\ 2i+2, & \frac{m}{2}+1 \leq i \leq m \text{ and i is odd} \\ 2i-2, & \frac{m}{2}+1 \leq i \leq m \text{ and i is even} \end{cases}$$

$$\text{and } f(v_i) = \begin{cases} 2m+2i-2, & 3 \leq i \leq \frac{n}{2} \text{ and i is odd} \\ 2m+2i-6, & 3 \leq i \leq \frac{n}{2} \text{ and i is even} \\ 2m+2i-2, & \frac{n}{2}+1 \leq i \leq n. \end{cases}$$

Then the induced edge labeling f^* is obtained as follows:

$$f^*(u_iu_{i+1}) = \begin{cases} 2i-1, & 1 \leq i \leq \frac{m}{2}-1 \\ 2i+1, & \frac{m}{2} \leq i \leq m-1, \end{cases}$$

$f^*(u_mu_1) = m - 1,$

$$f^*(v_iv_{i+1}) = \begin{cases} 2m+2i-3, & 2 \leq i \leq \frac{n}{2} \\ 2m+2i-1, & \frac{n}{2}+1 \leq i \leq n-1 \end{cases}$$

and $f^*(v_nv_1) = 2m + n - 1,$

Hence f is an even vertex odd mean labeling of $C_m \ominus C_n$.

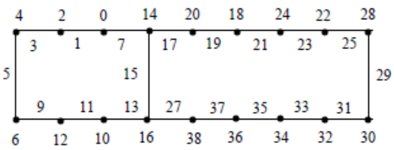

Figure 2: An even vertex odd mean labeling of $C_8 \ominus C_{12}$.

Case (ii). $m \equiv 2 \pmod 4$ and $n \equiv 0 \pmod 4$ or m, $n \equiv 2 \pmod 4$ with $m \leq n$.

Let the identifying edge be $u_{m-1}u_m$ and v_1v_n. Then the graph will be viewed as a cycle of length m+n−2, whose vertices are $z_1, z_2, \ldots, z_{m+n-2}$ with a chord $z_{m-1}z_{m+n-2}$.

Define $f : V(Cm \ominus Cn) \rightarrow \{0, 2, 4, \ldots, 2(m + n - 1)\}$ as follows:

$$f(z_i) = \begin{cases} 2i-2, & 1 \leq i \leq \frac{m+n}{2} \\ 2i+2, & \frac{m+n}{2}+1 \leq i \leq m+\frac{n}{2}-3 \text{ and i is even} \\ 2i-2, & \frac{m+n}{2}+1 \leq i \leq m+\frac{n}{2}-3 \text{ and i is odd} \\ 2i+2, & m+\frac{n}{2}-2 \leq i \leq m+n-2. \end{cases}$$

Then the induced edge labeling f^* is obtained as follows:

$$f^*(z_iz_{i+1}) = \begin{cases} 2i-1, & 1 \leq i \leq \frac{m+n}{2}-1 \\ 2i+1, & \frac{m+n}{2} \leq i \leq m+\frac{n}{2}-3 \\ 2i+3, & m+\frac{n}{2}-2 \leq i \leq m+n-3, \end{cases}$$

$f^*(z_1z_{m+n-2}) = m + n - 1$

and $f^*(z_{m-1}z_{m+n-2}) = 2m + n - 3,$

Thus f is an even vertex odd mean labeling of $C_m \ominus C_n$. □

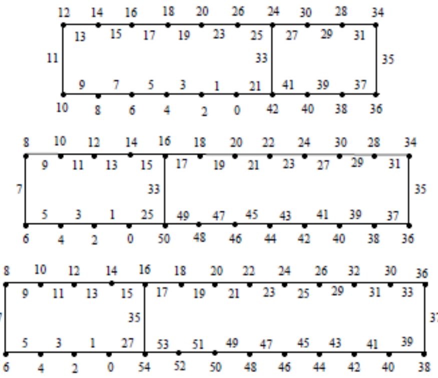

Figure 3: An even vertex odd mean labeling of $C_{14} \ominus C_8, C_{10} \ominus C_{16}$ and $C_{10} \ominus C_{18}$.

$P_n @ P_m$ is a graph obtained by identifying the pendant vertex of a copy of path P_m at each vertex of the path P_n.

Proposition 2.3. $P_n @ P_m$ is an even vertex odd mean graph for any n, $m \geq 1$.

Proof. Let $v_1, v_2, \ldots, v_n$ be the vertices on the path P_n. Let $v_{i,1}, v_{i,2}, \ldots, v_{i,m}$ be the vertices on the i^{th} copy of P_m so that $v_{i,1}$ is identified with v_i for $1 \leq i \leq n$.

The number of edges in $P_n @ P_m$ is mn − 1.

Define $f : V(P_n @ P_m) \rightarrow \{0, 2, 4, 6, \ldots, 2mn - 2\}$ as follows:

$$f(v_{i,j}) = \begin{cases} 2m(i-1)+2(j-1), & 1 \leq j \leq m, 1 \leq i \leq n \text{ and i is odd} \\ 2mi-2j, & 1 \leq j \leq m, 1 \leq i \leq n \text{ and i is even.} \end{cases}$$

Then the induced edge labeling f^* is obtained as follows:

$$f^*(v_{i,j}v_{i,j+1}) = \begin{cases} 2m(i-1)+2j-1, & 1 \leq j \leq m-1, 1 \leq i \leq n \text{ and i is odd} \\ 2mi-2j-1, & 1 \leq j \leq m-1, 1 \leq i \leq n \text{ and i is even} \end{cases}$$

and $f^*(v_{i,1}v_{i+1,1}) = 2mi - 1$, $1 \leq i \leq n - 1$.

Hence f is an even vertex odd mean labeling of $P_n @ P_m$. □

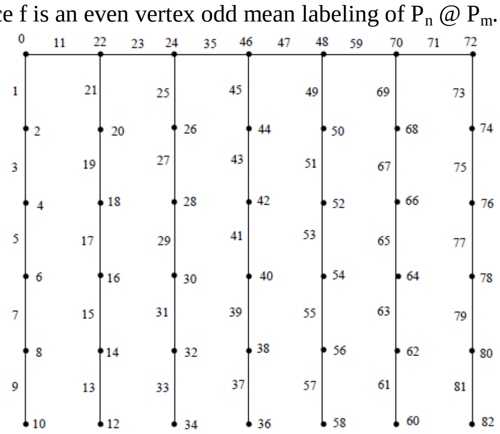

Figure 4: An even vertex odd mean labeling of $P_7 @ P_6$.

Corollary 2.4. $P_n \odot K_1$ is an even vertex odd mean graph for any $n \geq 1$.

The cycle quadrilateral snake CQ_n is a graph obtained from the cycle C_n by identifying each edge of C_n with an edge of C_4.

Theorem 2.5. Cycle Quadrilateral snake CQ_n is an even vertex odd mean graph if and only if n is even.

Proof. Let $v_1, v_2, \ldots, v_n$ be the vertices on the cycle C_n and let x_i, y_i, $1 \leq i \leq n$ be the vertices adjacent to v_i and v_{i+1} respectively, where $v_{n+1} = v_1$.

Case (i). $n \equiv 0 \pmod 4$.

Define $f : V(CQ_n) \to \{0, 2, 4, \dots, 8n\}$ as follows:

$$f(v_i) = \begin{cases} 8i-8, & 1 \le i \le \frac{n}{2}-1 \text{ and i is odd} \\ 8i, & \frac{n}{2} \le i \le n \text{ and i is odd} \\ 8i-6, & 1 \le i \le n \text{ and i is even,} \end{cases}$$

$$f(x_i) = \begin{cases} 8i-6, & 1 \le i \le n \text{ and i is odd} \\ 8i-8, & 1 \le i \le \frac{n}{2} \text{ and i is even} \\ 8i, & \frac{n}{2}+1 \le i \le n \text{ and i is even} \end{cases}$$

$$\text{and } f(y_i) = \begin{cases} 8i-4, & 1 \le i \le \frac{n}{2}-1 \text{ and i is odd} \\ 8i+4, & \frac{n}{2} \le i \le n \text{ and i is odd} \\ 8i-2, & 1 \le i \le n \text{ and i is even.} \end{cases}$$

Then the induced edge labeling f^* is obtained as follows:

$$f^*(v_iv_{i+1}) = \begin{cases} 8i-3, & 1 \le i \le \frac{n}{2}-1 \\ 8i+1, & \frac{n}{2} \le i \le n-1 \\ 4i-3, & i = n \end{cases}$$

$$f^*(x_iv_i) = \begin{cases} 8i-7, & 1 \le i \le \frac{n}{2}-1 \\ 8i-3, & \frac{n}{2} \le i \le n, \end{cases}$$

$$f^*(x_iy_i) = \begin{cases} 8i-5, & 1 \le i \le \frac{n}{2} \\ 8i-1, & \frac{n}{2}+1 \le i \le n, \end{cases}$$

$$f^*(y_iv_{i+1}) = \begin{cases} 8i-1, & 1 \le i \le \frac{n}{2}-1 \\ 8i+3, & \frac{n}{2} \le i \le n-1 \\ 4i-1, & i = n. \end{cases}$$

Thus f is an even vertex odd mean labeling of CQ_n.

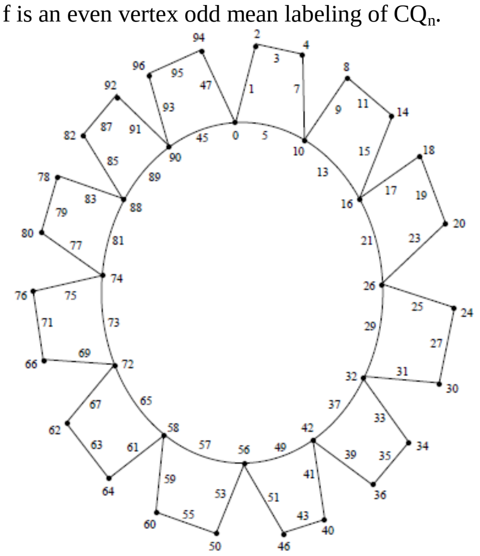

Figure 5: An even vertex odd mean labeling of CQ_{12}.

Case (ii). $n \equiv 2 \pmod 4$.

Define $f : V(CQ_n) \to \{0, 2, 4, \dots, 8n\}$ as follows:

$$f(v_i) = \begin{cases} 8i-8, & 1 \le i \le \frac{n}{2} \text{ and i is odd} \\ 8i, & \frac{n}{2}+1 \le i \le n \text{ and i is odd} \\ 8i-6, & 1 \le i \le \frac{n}{2}-1 \text{ and i is even} \\ 8i+2, & i = \frac{n}{2}+1 \\ 8i-6, & \frac{n}{2}+2 \le i \le n \text{ and i is even,} \end{cases}$$

$$f(x_i) = \begin{cases} 8i-6, & 1 \le i \le \frac{n}{2} \text{ and i is odd} \\ 8i-14, & i = \frac{n}{2}+2 \\ 8i-6, & \frac{n}{2}+3 \le i \le \frac{n}{2} \text{ and i is odd} \\ 8i-8, & 1 \le i \le \frac{n}{2}+1 \text{ and i is even} \\ 8i, & \frac{n}{2}+2 \le i \le n \text{ and i is even} \end{cases}$$

$$\text{and } f(y_i) = \begin{cases} 8i-4, & 1 \le i \le \frac{n}{2} \text{ and i is odd} \\ 8i+4, & \frac{n}{2}+1 \le i \le n \text{ and i is odd} \\ 8i-2, & 1 \le i \le \frac{n}{2}-1 \text{ and i is even} \\ 8i+6, & i = \frac{n}{2}+1 \\ 8i-2, & \frac{n}{2}+2 \le i \le n \text{ and i is even.} \end{cases}$$

Then the induced edge labeling f^* is obtained as follows:

$$f^*(v_iv_{i+1}) = \begin{cases} 8i-3, & 1 \le i \le \frac{n}{2}-1 \\ 8i+1, & i = \frac{n}{2} \\ 8i+5, & i = \frac{n}{2}+1 \\ 8i+1, & \frac{n}{2}+2 \le i \le n-1 \\ 4i-3, & i = n, \end{cases}$$

$$f^*(x_iv_i) = \begin{cases} 8i-7, & 1 \le i \le \frac{n}{2} \\ 8i-3, & i = \frac{n}{2}+1 \\ 8i-7, & i = \frac{n}{2}+2 \\ 8i-3, & \frac{n}{2}+3 \le i \le n, \end{cases}$$

$$f^*(x_iy_i) = \begin{cases} 8i-5, & 1 \le i \le \frac{n}{2} \\ 8i-1, & i = \frac{n}{2}+1 \\ 8i-5, & i = \frac{n}{2}+2 \\ 8i-1, & \frac{n}{2}+3 \le i \le n, \end{cases}$$

$$\text{and } f^*(y_iv_{i+1}) = \begin{cases} 8i-1, & 1 \le i \le \frac{n}{2}-1 \\ 8i+3, & i = \frac{n}{2} \\ 8i+7, & i = \frac{n}{2}+1 \\ 8i+3, & \frac{n}{2}+2 \le i \le n-1 \\ 4i-3, & i = n. \end{cases}$$

Thus f is an even vertex odd mean labeling of CQ_n. □

Theorem 2.6. Let G be a graph obtained from $P_n \odot K_1$ by identifying each edge with an edge of C_4. Then G is an even vertex odd mean graph, for all $n \ge 1$.

Proof. Let $v_1, v_2, \dots, v_n$ be the vertices on the path of length $n-1$ in G. Let u_iw_i, $1 \le i \le n-1$ be the edges of G so that $u_iv_i \in E(G)$ and $w_iv_{i+1} \in E(G)$ and $x_iy_iz_i$, $1 \le i \le n$ be the paths of length 2 so that x_iv_i, $z_iv_i \in E(G)$. The number of edges in G is $8n-4$. Assume that $n \ge 2$.

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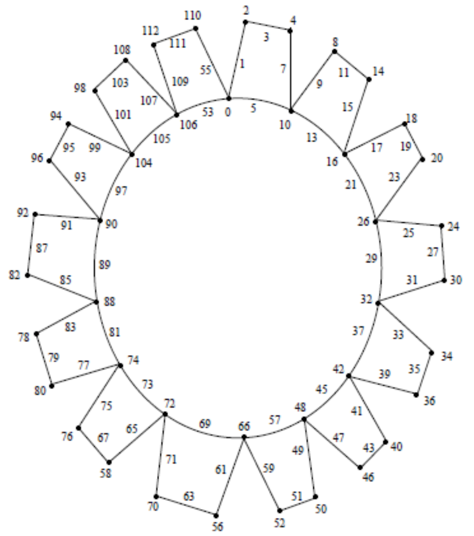

Figure 6: An even vertex odd mean labeling of CQ_{14}.

Define f : V (G) → {0, 2, 4, . . . , 16n − 8} as follows:

$$f(v_i) = \begin{cases} 16i-12, & 1 \le i \le n \text{ and i is odd} \\ 16i-14, & 1 \le i \le n \text{ and i is even,} \end{cases}$$

$$f(u_i) = \begin{cases} 16i-2, & 1 \le i \le n-1 \text{ and i is odd} \\ 16i, & 1 \le i \le n-1 \text{ and i is even,} \end{cases}$$

$$f(w_i) = \begin{cases} 16i-4, & 1 \le i \le n-1 \text{ and i is odd} \\ 16i-6, & 1 \le i \le n-1 \text{ and i is even,} \end{cases}$$

$$f(x_i) = \begin{cases} 2, & i = 1 \\ 16i-18, & 2 \le i \le n \text{ and i is odd} \\ 16i-16, & 2 \le i \le n \text{ and i is even} \end{cases}$$

$$f(y_i) = \begin{cases} 0, & i = 1 \\ 16i-8, & 2 \le i \le n \text{ and i is odd} \\ 16i-10, & 2 \le i \le n \text{ and i is even} \end{cases}$$

and
$$f(z_i) = \begin{cases} 10, & i = 1 \\ 16i-10, & 2 \le i \le n \text{ and i is odd} \\ 16i-8, & 2 \le i \le n \text{ and i is even.} \end{cases}$$

Then the induced edge labeling f^* is obtained as follows:

$f^*(v_iv_{i+1}) = 16i - 5, 1 \le i \le n - 1,$

$f^*(w_iv_{i+1}) = 16i - 1, 1 \le i \le n - 1,$

$f^*(u_iv_i) = 16i - 7, 1 \le i \le n - 1,$

$f^*(u_iw_i) = 16i - 3, 1 \le i \le n - 1,$

$$f^*(v_ix_i) = \begin{cases} 3, & i = 1 \\ 16i-15, & 2 \le i \le n, \end{cases}$$

$$f^*(x_iy_i) = \begin{cases} 1, & i = 1 \\ 16i-13, & 2 \le i \le n, \end{cases}$$

$$f^*(y_iz_i) = \begin{cases} 5, & i = 1 \\ 16i-9, & 2 \le i \le n \end{cases}$$

and
$$f^*(z_iv_i) = \begin{cases} 7, & i = 1 \\ 16i-11, & 2 \le i \le n. \end{cases}$$

Thus f is an even vertex odd mean labeling of G.

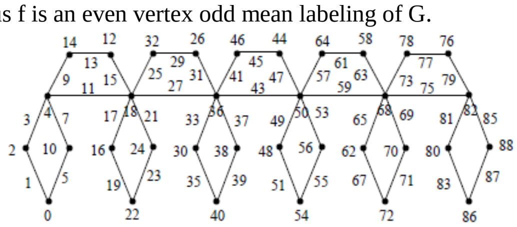

Figure 7: An even vertex odd mean labeling of G when $n = 6$

When n = 1, an even vertex odd mean labeling of G is given in Figure 8. □

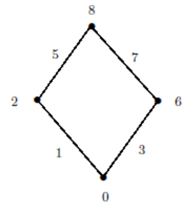

Figure 8. An even vertex odd mean labeling of G when $n = 1$.

Theorem 2.7. Let G be a graph obtained from $C_n \odot K_1$ by identifying each edge with an edge of C_4. Then G is an even vertex odd mean graph if and only if n is even.

Proof. Let $v_1, v_2, . . . , v_n$ be the vertices on the cycle and u_i, $1 \le i \le n$ be the pendant vertex attached at v_i in $C_n \odot K_1$. Let G be the graph obtained from $C_n \odot K_1$ by identifying an edge of a copy of C_4 in each edge of $C_n \odot K_1$.

Let $V(G) = \{u_i, v_i, w_i, x_i, y_i, z_i : 1 \le i \le n\}$ and $E(G) = \{v_iv_{i+1}, z_iv_{i+1} : 1 \le i \le n-1\} \cup \{v_1v_n, z_nv_1\} \cup \{u_iv_i, u_iw_i, w_ix_i, x_iv_i, y_iz_i, v_iy_i : 1 \le i \le n\}$.

Case (i). n ≡ 0(mod 4)

Define f : V (G) → {0, 2, 4, . . . , 16n} as follows:

$$f(v_i) = \begin{cases} 16i-8, & 1 \le i \le n \text{ and i is odd} \\ 16i-14, & 1 \le i \le \frac{n}{2} \text{ and i is even} \\ 16i-6, & \frac{n}{2}+1 \le i \le n \text{ and i is even,} \end{cases}$$

$$f(u_i) = \begin{cases} 2, & i = 1 \\ 16i-18, & 2 \le i \le \frac{n}{2} \text{ and i is odd} \\ 16i-10, & \frac{n}{2}+1 \le i \le n \text{ and i is odd} \\ 16i-12, & 2 \le i \le \frac{n}{2} \text{ and i is even,} \end{cases}$$

$$f(w_i) = \begin{cases} 0, & i = 1 \\ 16i-12, & 2 \le i \le n \text{ and i is odd} \\ 16i-18, & 2 \le i \le \frac{n}{2} \text{ and i is odd} \\ 16i-10, & \frac{n}{2}+1 \le i \le n \text{ and i is even,} \end{cases}$$

$$f(x_i) = \begin{cases} 16i-10, & 1 \le i \le \frac{n}{2} \text{ and i is odd} \\ 16i-2, & \frac{n}{2}+1 \le i \le n \text{ and i is odd} \\ 16i-4, & 1 \le i \le n \text{ and i is even,} \end{cases}$$

$$f(y_i) = \begin{cases} 16i-6, & 1 \le i \le \frac{n}{2} \text{ and i is odd} \\ 16i+2, & \frac{n}{2}+1 \le i \le n \text{ and i is odd} \\ 16i, & 1 \le i \le n \text{ and i is even} \end{cases}$$

and
$$f(z_i) = \begin{cases} 16i-4, & 1 \le i \le n \text{ and i is odd} \\ 16i-10, & 1 \le i \le \frac{n}{2} \text{ and i is even} \\ 16i-2, & \frac{n}{2}+1 \le i \le n \text{ and i is even.} \end{cases}$$

Then the induced edge labeling f^* is obtained as follows:

$$f^*(v_iv_{i+1}) = \begin{cases} 16i-3, & 1 \le i \le \frac{n}{2} \\ 16i+1, & \frac{n}{2}+1 \le i \le n-1, \end{cases}$$

$f^*(v_nv_1) = 8n + 1,$

$$f^*(v_iu_i) = \begin{cases} 8i-3, & i = 1 \\ 16i-13, & 2 \le i \le \frac{n}{2} \\ 16i-9, & \frac{n}{2}+1 \le i \le n, \end{cases}$$

$$f^*(u_iw_i) = \begin{cases} 16i-15, & 1 \le i \le \frac{n}{2} \\ 16i-11, & \frac{n}{2}+1 \le i \le n, \end{cases}$$

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$$f^*(w_ix_i) = \begin{cases} 3, & i = 1 \\ 16i-11, & 2 \le i \le \frac{n}{2} \\ 16i-7, & \frac{n}{2}+1 \le i \le n, \end{cases}$$

$$f^*(x_iv_i) = \begin{cases} 16i-9, & 1 \le i \le \frac{n}{2} \\ 16i-5, & \frac{n}{2}+1 \le i \le n, \end{cases}$$

$$f^*(v_iy_i) = \begin{cases} 16i-7, & 1 \le i \le \frac{n}{2} \\ 16i-3, & \frac{n}{2}+1 \le i \le n, \end{cases}$$

$$f^*(y_iz_i) = \begin{cases} 16i-5, & 1 \le i \le \frac{n}{2} \\ 16i-1, & \frac{n}{2}+1 \le i \le n, \end{cases}$$

$$f^*(z_iv_{i+1}) = \begin{cases} 16i-1, & 1 \le i \le \frac{n}{2} \\ 16i+3, & \frac{n}{2}+1 \le i \le n-1 \end{cases}$$

and $f^*(z_nv_1) = 8n + 3$.

Thus f is an even vertex odd mean labeling of G.

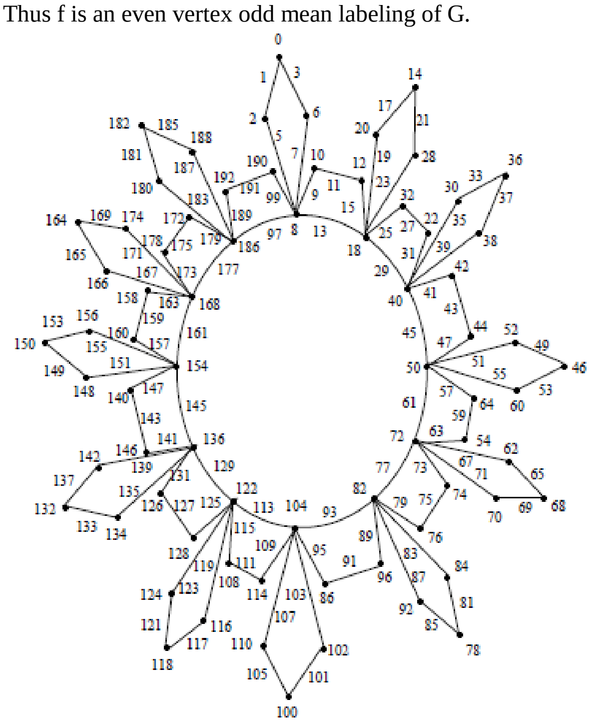

Figure 9: An even vertex odd mean labeling of G when $n = 12$

Case (ii). $n \equiv 2 \pmod 4$

Define $f : V(G) \to \{0, 2, 4, \ldots, 16n\}$ as follows:

$$f(v_i) = \begin{cases} 16i-8, & 1 \le i \le \frac{n}{2} \text{ and i is odd} \\ 16i, & \frac{n}{2}+1 \le i \le n \text{ and i is odd} \\ 16i-4, & 1 \le i \le n-1 \text{ and i is even} \\ 16i-6, & i = n, \end{cases}$$

$$f(u_i) = \begin{cases} 2, & i = 1 \\ 16i-18, & 2 \le i \le n \text{ and i is odd} \\ 16i-12, & 1 \le i \le \frac{n}{2} \text{ and i is even} \\ 16i-4, & \frac{n}{2}+1 \le i \le n-1 \text{ and i is even} \\ 16i-8, & i = n, \end{cases}$$

$$f(w_i) = \begin{cases} 0, & i = 1 \\ 16i-12, & 2 \le i \le \frac{n}{2} \text{ and i is odd} \\ 16i-4, & \frac{n}{2}+1 \le i \le n-1 \text{ and i is odd} \\ 16i-18, & 1 \le i \le n-1 \text{ and i is even} \\ 16i-22, & i = n, \end{cases}$$

$$f(x_i) = \begin{cases} 16i-10, & 1 \le i \le n \text{ and i is odd} \\ 16i-4, & 1 \le i \le \frac{n}{2} \text{ and i is even} \\ 16i+4, & \frac{n}{2}+1 \le i \le n-1 \text{ and i is even} \\ 16i-4, & i = n, \end{cases}$$

$$f(y_i) = \begin{cases} 16i-6, & 1 \le i \le n \text{ and i is odd} \\ 16i, & 1 \le i \le \frac{n}{2} \text{ and i is even} \\ 16i+8, & \frac{n}{2}+1 \le i \le n-1 \text{ and i is even} \\ 16i, & i = n, \end{cases}$$

$$\text{and } f(z_i) = \begin{cases} 16i-4, & 1 \le i \le \frac{n}{2} \text{ and i is odd} \\ 16i+4, & \frac{n}{2}+1 \le i \le n \text{ and i is odd} \\ 16i-10, & 1 \le i \le n-1 \text{ and i is even} \\ 16i-2, & i = n. \end{cases}$$

Then the induced edge labeling f^* is obtained as follows:

$$f^*(v_iv_{i+1}) = \begin{cases} 16i-3, & 1 \le i \le \frac{n}{2} \\ 16i+1, & \frac{n}{2}+1 \le i \le n-2 \\ 16i+5, & i = n-1, \end{cases}$$

$f^*(v_nv_1) = 8n + 1$,

$$f^*(v_iu_i) = \begin{cases} 5, & i = 1 \\ 16i-13, & 2 \le i \le \frac{n}{2} \\ 16i-9, & \frac{n}{2}+1 \le i \le n-1 \\ 16i-7, & i = n, \end{cases}$$

$$f^*(u_iw_i) = \begin{cases} 1, & i = 1 \\ 16i-15, & 2 \le i \le \frac{n}{2} \\ 16i-11, & \frac{n}{2}+1 \le i \le n-1 \\ 16i-15, & i = n, \end{cases}$$

$$f^*(w_ix_i) = \begin{cases} 3, & i = 1 \\ 16i-11, & 2 \le i \le \frac{n}{2} \\ 16i-7, & \frac{n}{2}+1 \le i \le n-1 \\ 16i-13, & i = n, \end{cases}$$

$$f^*(w_iv_i) = \begin{cases} 16i-9, & 1 \le i \le \frac{n}{2} \\ 16i-5, & \frac{n}{2}+1 \le i \le n, \end{cases}$$

$$f^*(v_iy_i) = \begin{cases} 16i-7, & 1 \le i \le \frac{n}{2} \\ 16i-3, & \frac{n}{2}+1 \le i \le n, \end{cases}$$

$$f^*(y_iz_i) = \begin{cases} 16i-5, & 1 \le i \le \frac{n}{2} \\ 16i-1, & \frac{n}{2}+1 \le i \le n \end{cases}$$

$$\text{and } f^*(z_iv_i) = \begin{cases} 16i-1, & 1 \le i \le \frac{n}{2} \\ 16i+3, & \frac{n}{2}+1 \le i \le n-1 \\ 16i+7, & i = n-1 \\ 8i+3, & i = n. \end{cases}$$

Thus f is an even vertex odd mean labeling of G. □

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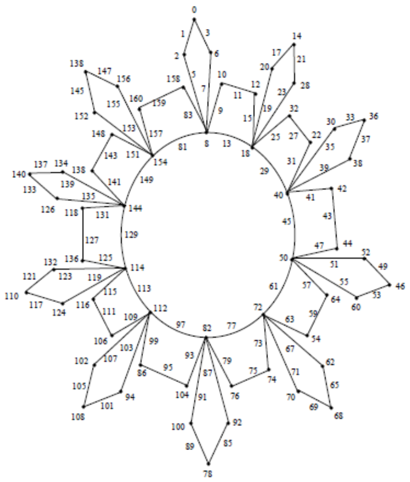

Figure 10: An even vertex odd mean labeling of G when $n = 10$.

Theorem 2.8. $P(m_1,m_2, \ldots ,m_{n-1})$ is the graph obtained by replacing each i^{th} edge of the path P_n by K_{2,m_i}, $1 \le i \le n-1$. Then $P(m_1,m_2, \ldots ,m_{n-1})$ is an even vertex odd mean graph for all $m_i \ge 1$, $1 \le i \le n-1$.

Proof. Let $v_1, v_2, \ldots, v_n$ be the vertices on the path P_n. Let $x_{i,1}, x_{i,2}, \ldots, x_{i,m_i}$ be the vertices of one of the partition of K_{2,m_i} which are adjacent to both v_i and v_{i+1}, $1 \le i \le n-1$.

Define $f : V(P(m_1,m_2, \ldots ,m_{n-1})) \to \{0, 2, 4, 6, \ldots, 4\sum_{i=1}^{n-1} m_i\}$ as follows:

$f(v_1) = 0$,

$f(v_i) = 4\sum_{j=1}^{i-1} m_j$, $2 \le i \le n$

and $f(x_{i,j}) = f(v_i) + 4j - 2$, $1 \le i \le n-1$, $1 \le j \le m_i$.

Then the induced edge labeling f^* is obtained as follows:

$f^*(v_i x_{i,j}) = f(v_i) + 2j - 1$, $1 \le i \le n-1$, $1 \le j \le m_i$

and $f^*(x_{i,j}v_{i+1}) = f(v_i) + 2m_i + 2j - 2$, $1 \le i \le n-1$, $1 \le j \le m_i$.

Thus f is an even vertex odd mean labeling of $P(m_1,m_2, \ldots, m_{n-1})$. □

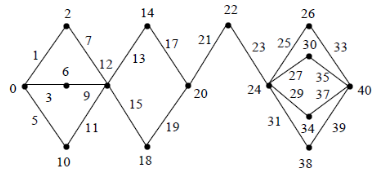

Figure 11: An even vertex odd mean labeling of $P(3, 2, 1, 4)$.

$D_2(P_n)$ is a graph obtained from two copies of P_n so that the pair of vertices u, v' and u', v are adjacent in $D_2(P_n)$ whenever uv and $u'v'$ are the respective edges of the copies of P_n.

Theorem 2.9. For any $n \ge 2$, $D_2(P_n)$ is an even vertex odd mean graph.

Proof. Let $u_1, u_2, \ldots, u_n$ and $v_1, v_2, \ldots, v_n$ be the vertices on the paths of length $n-1$ in $D_2(P_n)$.

Case (i). $n \ge 3$ and n is odd.

Define $f : V(D_2(P_n)) \to \{0, 2, 4, \ldots, 8(n-1)\}$ as follows:

$$f(u_i) = \begin{cases} 2i, & i = 1 \\ 0, & i = 2 \\ 8i - 18, & 3 \le i \le n \text{ and i is odd} \\ 8i - 4, & 3 \le i \le n \text{ and i is even,} \end{cases}$$

$$\text{and } f(v_i) = \begin{cases} 10, & i = 1 \\ 8i, & 2 \le i \le n \text{ and i is even} \\ 8i - 10, & 2 \le i \le n \text{ and i is odd.} \end{cases}$$

Then the induced edge labeling f^* is obtained as follows:

$$f^*(u_iu_{i+1}) = \begin{cases} 2i - 1, & i = 1, 2 \\ 5i + 2, & i = 3 \\ 8i + 7, & 4 \le i \le n-1, \end{cases}$$

$$f^*(v_iv_{i+1}) = \begin{cases} 4i + 9, & i = 1 \\ 8i - 1, & 2 \le i \le n-1, \end{cases}$$

$$f^*(u_iv_{i+1}) = \begin{cases} 5i + 8, & i = 1 \\ 4i - 1, & i = 2 \\ 8i - 5, & 3 \le i \le n-1 \text{ and i is odd} \\ 8i - 3, & 3 \le i \le n-1 \text{ and i is even} \end{cases}$$

$$\text{and } f^*(v_iu_{i+1}) = \begin{cases} 5, & i = 1 \\ 8i - 3, & 2 \le i \le n-1 \text{ and i is odd} \\ 8i - 5, & 2 \le i \le n-1 \text{ and i is even.} \end{cases}$$

Thus f is an even vertex odd mean labeling of $D_2(P_n)$.

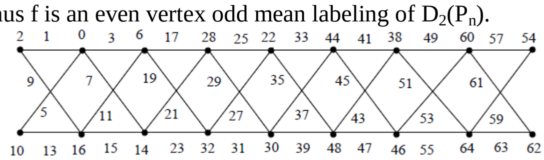

Figure 12: An even vertex odd mean labeling of $D_2(P_9)$.

Case (ii). $n \ge 2$ and n is even.

Define $f : V(D_2(P_n)) \to \{0, 2, 4, \ldots, 8(n-1)\}$ as follows:

$$f(u_i) = \begin{cases} 8i - 8, & 1 \le i \le n \text{ and i is odd} \\ 8i - 14, & 1 \le i \le n \text{ and i is even} \end{cases}$$

$$\text{and } f(v_i) = \begin{cases} 8i, & 1 \le i \le n \text{ and i is odd} \\ 8i - 10, & 1 \le i \le n \text{ and i is even.} \end{cases}$$

Then the induced edge labeling f^* is obtained as follows:

$f^*(u_iu_{i+1}) = 8i - 7$, $1 \le i \le n-1$,

$f^*(v_iv_{i+1}) = 8i - 1$, $1 \le i \le n-1$,

$$f^*(u_iv_{i+1}) = \begin{cases} 8i - 5, & 1 \le i \le n-1 \text{ and i is odd} \\ 8i - 3, & 1 \le i \le n-1 \text{ and i is even} \end{cases}$$

$$\text{and } f^*(v_iu_{i+1}) = \begin{cases} 8i - 3, & 1 \le i \le n-1 \text{ and i is odd} \\ 8i - 5, & 1 \le i \le n-1 \text{ and i is even.} \end{cases}$$

Thus f is an even vertex odd mean labeling of $D_2(P_n)$. □

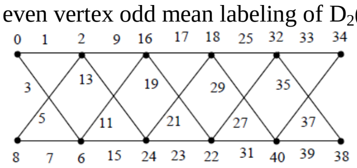

Figure 13: An even vertex odd mean labeling of $D_2(P_6)$.

The Splitting graph SP(G) is a graph obtained from G by adding a new vertex v' to each vertex v of G and joining v' with all the adjacent vertices of v in G.

Theorem 2.10. $SP(K_{1,n})$ is an even vertex odd mean graph only when n = 1 and 2.

Proof. The graph $SP(K_{1,n})$ is isomorphic to a graph obtained by identifying the central vertex of $K_{1,n}$ with a vertex of degree n in $K_{2,n}$. Let u be the central vertex and $v_1, v_2, \ldots, v_n$ be the pendant vertices of $K_{1,n}$. Let x be the duplicating vertex of u and $y_1, y_2, \ldots, y_n$ be the duplicating vertices of $v_1, v_2, \ldots, v_n$ respectively. Then the number of edges in $SP(K_{1,n})$ is 3n. If f is an even vertex odd mean labeling of $SP(K_{1,n})$, then the vertices of $SP(K_{1,n})$ are assigned by the labels {0, 2, 4,

. . . , 6n}. Based on the adjacency in $SP(K_{1,n})$, u and u′ are assigned by the labels congruent to 0(mod 4) (or 2(mod 4)) and the remaining vertices are assigned by the labels congruent to 2(mod 4) (or 0(mod 4)). In {0, 2, 4, . . . , 6n}, at most $\frac{3n}{2}+1$ numbers only be either 0(mod 4) or 2(mod 4) when n is even. But 2n vertices are to be labeled with same suit either 0(mod 4) or 2(mod 4). So the vertex labeling is possible only when $\frac{3n}{2}+1$= 2n, which is true only when n = 2. In {0, 2, 4, . . . , 6n}, at most $\frac{3n+1}{2}$ only be either 0(mod 4) or 2(mod 4), when n is odd. Also $\frac{3n+1}{2}$ = 2n is true only when n = 1.

An even vertex odd mean labeling of $SP(K_{1,n})$ when n = 1, 2, is given in Figure 14. Hence the result follows. □

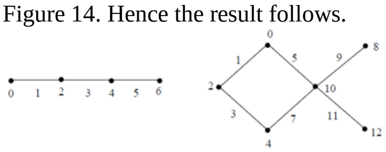

Figure 14: An even vertex odd mean labeling of $SP(K_{1,n})$ for $n = 1$ and 2.

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All material on each page should fit within a rectangle of 18 x 23.5 cm (7" x 9.25"), centered on the page, beginning 2.54 cm (1") from the top of the page and ending with 2.54 cm (1") from the bottom. The right and left margins should be 1.9 cm (.75"). The text should be in two 8.45 cm (3.33") columns with a .83 cm (.33") gutter.

3. TYPESET TEXT

3.1 Normal or Body Text

Please use a 9-point Times Roman font, or other Roman font with serifs, as close as possible in appearance to Times Roman in which these guidelines have been set. The goal is to have a 9-point text, as you see here. Please use sans-serif or non-proportional fonts only for special purposes, such as distinguishing source code text. If Times Roman is not available, try the font named Computer Modern Roman. On a Macintosh, use the font named Times. Right margins should be justified, not ragged.

3.2 This paragraph is a repeat of 3.1

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3.4 Title and Authors

The title (Helvetica 18-point bold), authors' names (Helvetica 12-point) and affiliations (Helvetica 10-point) run across the full width of the page – one column wide. We also recommend e-mail address (Helvetica 12-point). See the top of this page for three addresses. If only one address is needed, center all address text. For two addresses, use two centered tabs, and so on. For three authors, you may have to improvise.

3.5 Subsequent Pages

For pages other than the first page, start at the top of the page, and continue in double-column format. The two columns on the last page should be as close to equal length as possible.

Table 1. Table captions should be placed above the table

Graphics	Top	In-between	Bottom
Tables	End	Last	First
Figures	Good	Similar	Very well

3.6 Page Numbering, Headers and Footers

Do not include headers, footers or page numbers in your submission. These will be added when the publications are assembled.

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Place Tables/Figures/Images in text as close to the reference as possible (see Figure 1). It may extend across both columns to a maximum width of 17.78 cm (7").

Captions should be Times New Roman 9-point bold. They should be numbered (e.g., "Table 1" or "Figure 2"), please note that the word for Table and Figure are spelled out. Figure's captions should be centered beneath the image or picture, and Table captions should be centered above the table body.

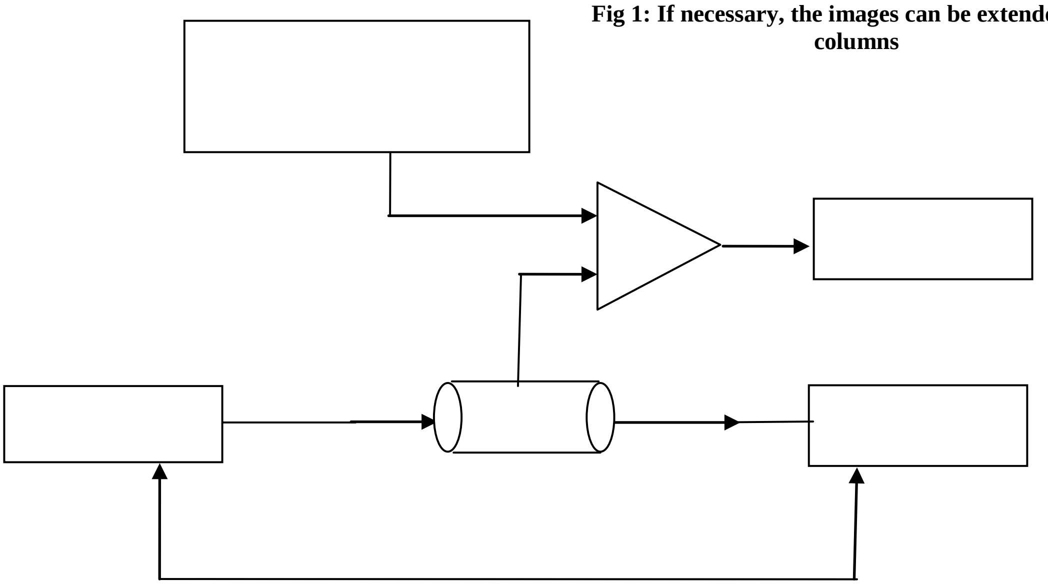

Fig 1: If necessary, the images can be extended both columns

5. SECTIONS

The heading of a section should be in Times New Roman 12-point bold in all-capitals flush left with an additional 6-points of white space above the section head. Sections and subsequent sub- sections should be numbered and flush left. For a section head and a subsection head together (such as Section 3 and subsection 3.1), use no additional space above the subsection head.

5.1 Subsections

The heading of subsections should be in Times New Roman 12-point bold with only the initial letters capitalized. (Note: For subsections and subsubsections, a word like *the* or*a* is not capitalized unless it is the first word of the header.)

5.1.1 Subsubsections

The heading for subsubsections should be in Times New Roman 11-point italic with initial letters capitalized and 6-points of white space above the subsubsection head.

5.1.1.1 Subsubsections

The heading for subsubsections should be in Times New Roman 11-point italic with initial letters capitalized.

5.1.1.2 Subsubsections

The heading for subsubsections should be in Times New Roman 11-point italic with initial letters capitalized.

6. ACKNOWLEDGMENTS

Our thanks to the experts who have contributed towards development of the template.

7. REFERENCES

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